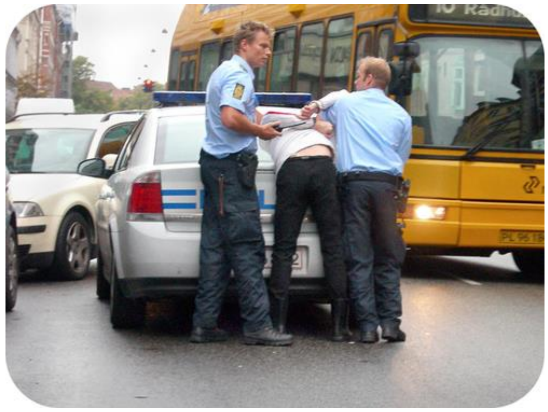
Uncertainty in Measurement

How do police officers identify criminals?



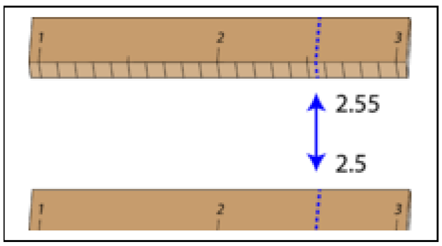
After a bank robbery has been committed, police will ask witnesses to describe the robbers. They will usually get some answer such as “medium height.” Others may say “between 5 foot 8 inches and 5 foot 10 inches.” In both cases, there is a significant amount of uncertainty about the height of the criminals.

Measurement Uncertainty

There are two types of numbers in the scientific world, exact numbers and inexact numbers. Exact numbers are numbers whose values are known exactly. For example, there are 12 in a dozen and 1000 grams in 1 kg.

Inexact numbers have values with some uncertainty. If you give 10 students each a dime and tell them to use a triple beam balance to obtain the mass of their dime, you will slightly varying masses. The reason for the differences may be due to equipment error, the balances not being calibrated equally, or human error, reading the balance wrong. Uncertainties always exist in measured quantities. The amount of uncertainty depends both upon the skill of the measurer and upon the quality of the measuring tool. While some balances are capable of measuring masses only to the nearest 0.1 g, other highly sensitive balances are capable of measuring to the nearest 0.001 g or even better. Many measuring tools such as rulers and graduated cylinders have small lines which need to be carefully read in order to make a measurement.

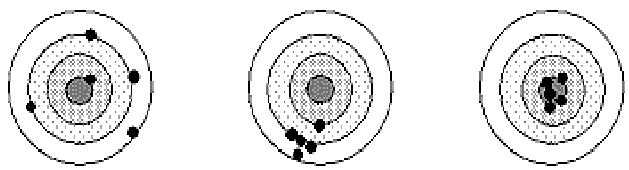
The figure to the left shows two rulers making the same measurement of an object (indicated by the arrow). With either ruler, it is clear that the length of the object is between 2 and 3 cm. The bottom ruler contains no millimeter markings. With that ruler, the tenths digit can be estimated and the length may be reported as 2.5 cm. However, another person may judge that the measurement is 2.4 cm or perhaps 2.6 cm. While the 2 is known for certain, the value of the tenths digit is uncertain.



The top ruler contains marks for tenths of a centimeter (millimeters). Now the same object may be measured as 2.55 cm. The measurer is capable of estimating the hundredths digit because he can be certain that the tenths digit is a 5. Again, another measurer may report the length to be 2.54 cm or 2.56 cm. In this case, there are two certain digits (the 2 and the 5), with the hundredths digit being uncertain. Clearly, the top ruler is a superior ruler for measuring lengths as precisely as possible.

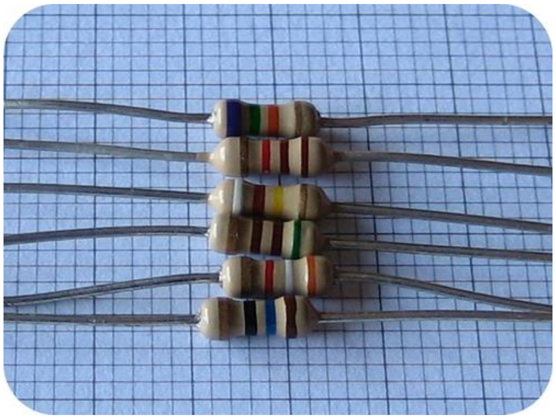
Precision and Accuracy

The terms precision and accuracy are often used in discussing the uncertainties of measured values. **Precision** is the measure of how closely individual measurements agree with one another. **Accuracy** refers to how closely individual measurements agree with the correct or “true” value. Refer to figure above for a visual representation of precision and accuracy.



Uncertainty in Measurement

Percent Error



How does an electrical circuit work?
A complicated piece of electronics equipment may contain several resistors whose role is to control the voltage and current in the electrical circuit. Too much current and the apparatus malfunctions. Too little current and the system simply doesn't perform. The resistors values are always given with an error range. A resistor may have a stated value of 200 ohms, but a 10% error range, meaning the resistance could be anywhere between 195-205 ohms. By knowing these values, an electronics person can design and service the equipment to make sure it functions properly.

Percent Error

An individual measurement may be accurate or inaccurate, depending on how close it is to the true value. Suppose that you are doing an experiment to determine the density of a sample of aluminum metal. The **accepted value** of a measurement is the true or correct value based on general agreement with a reliable reference. For aluminum the accepted density is 2.70g/cm^3. The **experimental value** of a measurement is the value that is measured during the experiment. Suppose that in your experiment you determine an experimental value for the aluminum density to be 2.42 g/cm^3. The **error** of an experiment is the difference between the experimental and accepted values.

$$\text{Error} = \text{experimental value} - \text{accepted value}$$

If the experimental value is less than the accepted value, the error is negative. If the experimental value is larger than the accepted value, the error is positive. Often, error is reported as the absolute value of the difference in order to avoid the confusion of a negative error. The **percent error** is the absolute value of the error divided by the accepted value and multiplied by 100%.

$$\%\ \text{Error} = \frac{|\text{experimental value} - \text{accepted value}|}{\text{accepted value}} \times 100\%$$

If the experimental value is equal to the accepted value, the percent error is equal to 0. As the accuracy of a measurement decreases, the percent error of that measurement rises.

Significant Figures

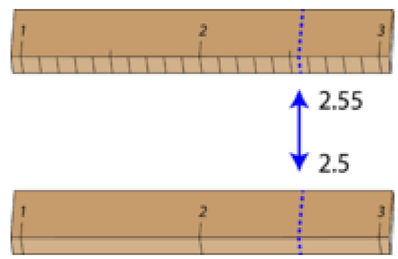
How fast do you drive?

Uncertainty in Measurement

As you enter the town of Jacinto City, Texas, the sign below tells you that the speed limit is 30 miles per hour. But what if you happen to be driving 31 miles an hour? Are you in trouble? Probably not, because there is a certain amount of leeway built into enforcing the regulation. Most speedometers do not measure the vehicle speed very accurately and could easily be off by a mile or so (on the other hand, radar measurements are much more accurate). So, a couple of miles/hour difference won't matter that much. Just don't try to stretch the limits any further unless you want a traffic ticket.

Significant Figures

The **significant figures** in a measurement consist of all the certain digits in that measurement plus one uncertain or estimated digit. In the ruler illustration below, the bottom ruler gives a length with 2 significant figures, while the top ruler gives a length with 3 significant figures. In a correctly reported measurement, the final digit is significant but not certain. Insignificant digits are not reported. With either ruler, it would not be possible to report the length as 2.553 cm as there is no possible way that the thousandths digit could be estimated. The 3 is not significant and would not be reported.



When you look at a reported measurement, it is necessary to be able to count the number of significant figures. The table below details the rules for determining the number of significant figures in a reported measurement. For the examples in the table below, assume that the quantities are correctly reported values of a measured quantity.

Significant Figure Rules

Rule	Examples
1. All nonzero digits in a measurement are significant	A. 237 has three significant figures. B. 1.897 has four significant figures.
2. Zeroes that appear between other nonzero digits are always significant.	A. 39,004 has five significant figures. B. 5.02 has three significant figures
3. Zeroes that appear in front of all of the nonzero digits are called left-end zeroes. Left-end zeroes are never significant	A. 0.008 has one significant figure. B. 0.000416 has three significant figures.
4. Zeroes that appear after all nonzero digits are called right-end zeroes. Right-end zeroes in a number that lacks a decimal point are not significant.	A. 140 has two significant figures. B. 75,210 has four significant figures.
5. Right-end zeroes in a number with a decimal point are significant. This is true	A. 620.0 has four significant figures. B. 19.000 has five significant figures

whether the zeroes occur before or after the decimal point.	

It needs to be emphasized that to say a certain digit is not significant does not mean that it is not important or can be left out. Though the zero in a measurement of 140 may not be significant, the value cannot simply be reported as 14. An insignificant zero functions as a placeholder for the decimal point. When numbers are written in scientific notation, this becomes more apparent. The measurement 140 can be written as 1.4×10^2 with two significant figures in the coefficient. For a number with left-end zeroes, such as 0.000416, it can be written as 4.16×10^{-4} with 3 significant figures. In some cases, scientific notation is the only way to correctly indicate the correct number of significant figures. In order to report a value of 15,000,000 with four significant figures, it would need to be written as 1.500×10^7. The right-end zeroes after the 5 are significant. The original number of 15,000,000 only has two significant figures.

Adding and Subtraction Significant Figures

For addition and subtraction, look at the decimal portion (i.e., to the right of the decimal point) of the numbers ONLY. Here is what to do:

1. Count the number of significant figures in the decimal portion of each number in the problem. (The digits to the left of the decimal place are not used to determine the number of decimal places in the final answer.)
2. Add or subtract in the normal fashion.
3. Round the answer to the LEAST number of places in the decimal portion of any number in the problem.

Multiplying and Dividing Significant Figures

The following rule applies for multiplication and division:

1. The LEAST number of significant figures in any number of the problem determines the number of significant figures in the answer.
2. This means you MUST know how to recognize significant figures in order to use this rule.

Resources

Video: Crash Course #2 - Significant Figures
https://www.youtube.com/watch?v=hQpQ0hxVNTg&list=PL8dPuuaLjXtPHzzYuWy6fYEaX9mQQ8oGr&index=3

Start at 7:40 (Significant Figures)