

NOTES 11.5b Translations of Conics

Parabolas

1. "Squared" variable on one side
2. Complete the square--coef of 1
3. Finish

All others

1. "Clump" the x's and y's
2. Complete the squares--coef of 1
3. Finish

Ex #1: Find the vertex and focus of the parabolas given by the equation. Sketch

$$x^2 - 2x + 4y - 3 = 0$$

$$\begin{aligned} V: (1, 1) &\leftarrow h, k \\ F: (1, 0) &\leftarrow h, k+p \\ D: y=2 &\leftarrow y=k-p \end{aligned}$$

$$x^2 - 2x + \underline{1} = -4y + 3 + \underline{1}$$

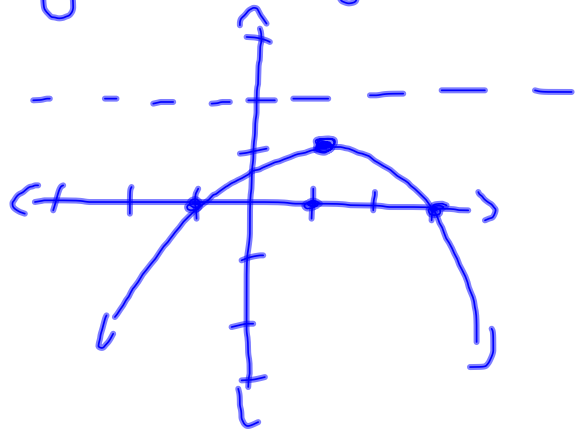
$$\frac{-1}{2} \quad (x-1)^2 = -4y + 4$$

$$\frac{-1}{(-1)^2} \quad (x-1)^2 = -4(y-1)$$

$$h = 1$$

$$k = 1$$

$$p = -1$$



Ex #2: Identify the center and radius of the circle given by the equation. Sketch

$$4x^2 + 4y^2 + 20x - 16y + 37 = 0$$

$$4x^2 + 20x + 4y^2 - 16y = -37$$

$$4\left(x^2 + 5x + \frac{25}{4}\right) + 4\left(y^2 - 4y + \frac{4}{1}\right) = -37 + \underline{25} + \underline{16}$$

$$4\left(x + \frac{5}{2}\right)^2 + 4(y - 2)^2 = 4$$

$$\left(x + \frac{5}{2}\right)^2 + (y - 2)^2 = 1$$

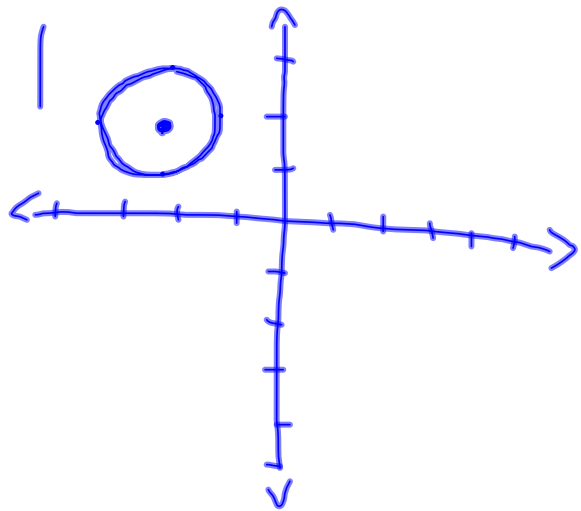
$$h = -\frac{5}{2}$$

$$k = 2$$

$$r = 1$$

$$C: \left(-\frac{5}{2}, 2\right)$$

$$r: 1$$



Ex #3: Identify the vertices and foci of the ellipse.
Sketch.

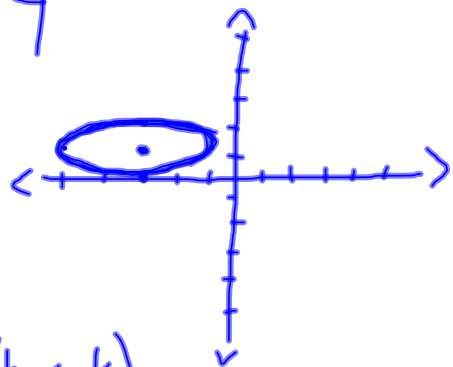
$$x^2 + 4y^2 + 6x - 8y + 9 = 0$$

$$x^2 + 6x + 4y^2 - 8y = -9$$

$$1(x^2 + 6x + \underline{9}) + 4(y^2 - 2y + \underline{1}) = -9 + \underline{9} + \underline{4}$$

$$1(x+3)^2 + 4(y-1)^2 = 4$$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{1} = 1$$



$$\left. \begin{array}{l} h: -3 \\ k: 1 \\ a: 2 \\ b: 1 \\ c: \sqrt{3} \\ 4-1=3 \end{array} \right\}$$

$$\begin{aligned} \bullet C: & (-3, 1) \quad (h, k) \\ V: & (-1, 1) \quad (5, 1) \quad (h+a, k) \quad (h-a, k) \\ F: & (-3+\sqrt{3}, 1) \quad (h+c, k) \quad (h-c, k) \\ & (-3-\sqrt{3}, 1) \end{aligned}$$

Ex #4: Identify the vertices and foci of the hyperbola. Sketch.

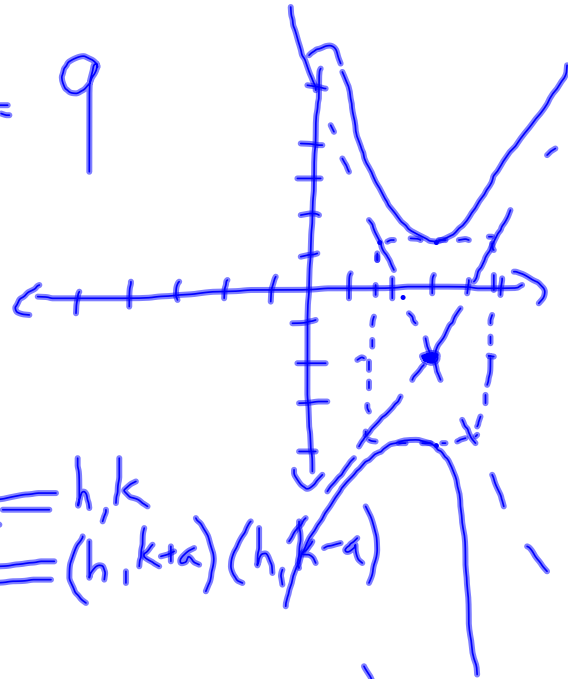
$$y^2 - 4x^2 + 4y + 24x - 41 = 0$$

$$y^2 + 4y \quad -4x^2 + 24x \quad = 41$$

$$1(y^2 + 4y + 4) - 4(x^2 - 6x + 9) = 41 + 4 - 36$$

$$1(y+2)^2 - 4(x-3)^2 = 9$$

$$\frac{(y+2)^2}{9} - \frac{(x-3)^2}{\frac{9}{4}} = 1$$



$$h = 3$$

$$k = -2$$

$$a = 3$$

$$b = \frac{3}{2}$$

$$c = \frac{3\sqrt{5}}{2}$$

$$9 + \frac{9}{4}$$

$$\frac{36}{4} + \frac{9}{4}$$

$$\frac{45}{4}$$

$$C : (3, -2) \leftarrow h, k$$

$$V : (3, 1) (3, -5) \leftarrow (h, k+a) (h, k-a)$$

$$F : \left(3, -2 + \frac{3\sqrt{5}}{2} \right) \left(3, -2 - \frac{3\sqrt{5}}{2} \right) \leftarrow (h, k+c) (h, k-c)$$